ARTICLE VII.

A Remarkable Arrangement of Numbers, constituting a Magic Cyclovolute. By E. Nulty, Philadelphia. Read before the American Philosophical Society, June 27th, 1834.

THE Magic Circle of Dr Franklin has been long admired, as embracing the most ingenious arrangement of numbers ever formed. It consists of *five* sets of circles, of which the first or principal includes *nine* circumferences, bounding *eight* concentric rings. These rings are equally intersected by *four* diameters or *eight* radii, on which, and in the middle of each ring, are placed the series of integral numbers from 12 to 75, both inclusive. In addition to this series, there is an auxiliary 12 occupying the common centre of the rings; and the total sixty-five numbers thus disposed, have, as respects the eight rings and eight radii, the following remarkable properties.

First. The eight numbers round each ring, with the auxiliary or central number, amount to 360, the number of sexagesimal degrees in a circle.

Secondly. The eight numbers along each radius, with the auxiliary number, amount to 360.

Thirdly. The four numbers in each semi-ring terminating in a principal diameter, intermediate between two particular radii, with half the auxiliary number, form the sum 180, the degrees in a semicircle.

Fourthly. Every four adjacent numbers in any two consecutive rings, with half the auxiliary number, give the same amount, 180.

As to the four remaining sets of circles and the rings which they form, their centres are at the four points in which the principal diameter, and a conjugate perpendicular to it, intersect the least and interior circumference. If our attention, for the instant, be confined to any one of these centres, and to the corresponding set of circles, the bounding circumferences of the exterior and interior rings will be seen to touch the greatest and least of the nine principal circumferences, at points in the principal diameter or its conjugate. According to this construction there are *five* rings between the bounding circumferences of each of the four sets of circles under consideration; and all the *twenty* rings thus constituted possess the same property with the eight rings first mentioned; or in more specific terms, the eight numbers in each of the twenty secondary rings, with the auxiliary number at the principal centre, form the sum 360.

These are the different properties comprised in the Magic Circle, left by its original and sagacious author. They certainly must be regarded as not a little curious, and would seem to require a considerable familiarity with the powers of numbers. As to the mode of investigation by which they were first discovered, we have seen no account sufficient to enable us to pronounce with any degree of confidence. We should not, however, be inclined to think that they resulted either from conjecture or trial, although they are by no means confined to the particular distribution of numbers published. We should rather be disposed to join in the opinion that they were suggested by remarks made on other arrangements previously formed. But still we are forced to believe that they must have been deduced from views which were incapable of embracing in its full extent the general problem, whence originated the present observations. The reasons which justify this conclusion will immediately appear on a glance at the drawing which accompanies this paper, and which may be regarded as a generalization of Dr Franklin's Magic Circle. The additions made are *Volutes*, commencing at the extremities of the diameters between the numbered radii; and on which account the drawing may not inappropriately be termed a *Magic Cyclovolute*.

To trace one of these curves, commence at the extremity A of the principal diameter AA′, and continue along the circle, of which the centre is a, nearly to the extent of a semicircle; then incline towards the least interior circle, aa' bb', and terminate in its circumference. In like manner another volute may be traced in the opposite direction, and thus will appear *two* of the volutes originating in the point A. *Six* similar volutes may be traced from the extreme points A′, B, B′; and all the *eight* viewed in pairs may be easily recognized by the four different colours in which they are delineated. Besides these volutes, we may trace *eight* analogous curves, from the extremities of the diameters intermediate between the conjugates AA′, BB′. In the drawing they may be traced by passing along circular segments, decreasing and changing their colours, whilst verging towards the interior circumference aa' bb'. There will thus appear *sixteen* similar volutes, in addition to the circles first described; and all these have precisely the same property relatively to the number 360, which forms the common result of the auxiliary 12, and every eight numbers within any two consecutive boundaries.

These, we believe, are all the properties of which the arrangement of numbers constituting the cyclovolute appears susceptible; and we intended to subjoin here the investigation which led to them, and to the different changes that may be made in disposing the numbers in the drawing. We have, however, concluded to omit this investigation for the moment, and make it the subject of a supplementary note to be read at a future meeting.

Regarding the objects of the Society, this paper is presented without any desire for its publication, and chiefly in compliance with the wishes of a friend. But as the Magic Circle originated, and has, I presume, been completed in Philadelphia; and as it has been considered in Europe as the most ingenious arrangement of numbers ever imagined, the Society may not be disinclined to insert some notice of the subject in their records.

MAGIC CYCLOVOLUTE.

Secondary Circles.

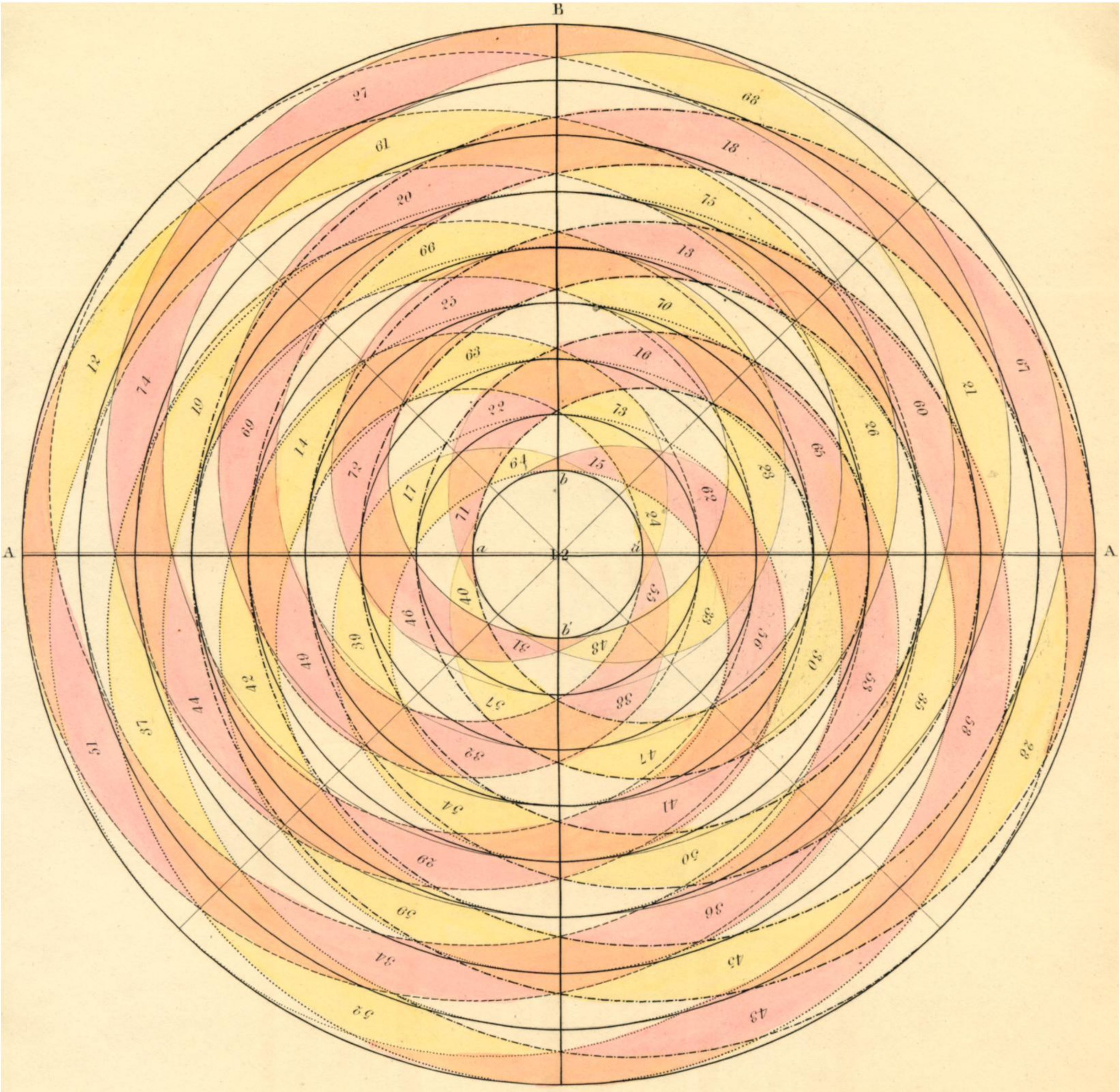
A					A′					B					B′				
51	37	44	42	49	67	21	60	26	65	27	61	20	66	25	43	45	36	50	41
34	59	29	54	32	18	75	13	70	16	74	19	69	14	72	58	35	53	30	56
36	50	41	47	38	20	66	25	63	22	44	42	49	39	46	60	26	65	23	62
53	30	56	33	55	69	14	72	17	71	29	54	32	57	31	13	70	16	73	15
26	65	23	62	24	42	49	39	46	40	50	41	47	38	48	66	25	63	22	64
75	13	70	16	73	59	29	54	32	57	35	53	30	56	33	19	69	14	72	17
61	20	66	25	63	45	36	50	41	47	21	60	26	65	23	37	44	42	49	39
12	74	19	69	14	28	58	35	53	30	68	18	75	13	70	52	34	59	29	54
12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12

360

Volutes.

A		A′		B		B′		A, B		A′, B		A, B′		A′, B′	
51	12	67	28	27	68	43	52	12	27	68	67	52	51	28	43
34	61	18	45	74	21	58	37	37	18	61	58	45	74	21	34
36	75	20	59	44	35	60	19	59	60	19	36	35	20	75	44
53	26	69	42	29	50	13	66	50	53	42	29	26	13	66	69
65	30	49	14	41	54	25	70	30	41	54	49	70	65	14	25
16	47	32	63	56	39	72	23	23	32	47	72	63	56	39	16
22	57	38	73	62	17	46	33	73	46	33	22	17	38	57	62
71	40	55	24	15	64	31	48	64	71	24	15	40	31	48	55
12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12

360



MAGIC CYCLOVOLUTE by E. NULTY.